

The Euler Identity:

$$e^{j\theta} = \cos \theta + j \sin \theta \quad (1)$$

where

$$j = \sqrt{-1} \quad (2)$$

Note that a consequence of the Euler identity is that

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad (3)$$

and

$$\sin \theta = \frac{je^{-j\theta} - je^{j\theta}}{2} \quad (4)$$

If you are curious, you can verify these fairly quickly by plugging (1) into the appropriate spots in (3) and (4).

With the Euler identity you can easily prove the trigonometric identity

$$(\cos \theta_1)(\cos \theta_2) = \frac{1}{2}(\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)) \quad (5)$$

This can be done by directly substituting (3) into the right-hand side of (5) and simplifying:

$$(\cos \theta_1)(\cos \theta_2) = \frac{e^{j\theta_1} + e^{-j\theta_1}}{2} \frac{e^{j\theta_2} + e^{-j\theta_2}}{2} \quad (6)$$

carrying out the multiplication, we get

$$(\cos \theta_1)(\cos \theta_2) = \frac{1}{4}(e^{j(\theta_1 + \theta_2)} + e^{j(-\theta_1 + \theta_2)} + e^{j(\theta_1 - \theta_2)} + e^{-j(\theta_1 + \theta_2)}) \quad (7)$$

Now observe that

$$\frac{1}{4}(e^{j(\theta_1 + \theta_2)} + e^{j(-\theta_1 + \theta_2)} + e^{j(\theta_1 - \theta_2)} + e^{-j(\theta_1 + \theta_2)}) = \frac{1}{2} \left(\frac{e^{j(\theta_1 + \theta_2)} + e^{-j(\theta_1 + \theta_2)}}{2} + \frac{e^{j(\theta_1 - \theta_2)} + e^{-j(\theta_1 - \theta_2)}}{2} \right) \quad (8)$$

By inspection, we can apply (3) to get

$$(\cos \theta_1)(\cos \theta_2) = \frac{1}{2}(\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)) \quad (9)$$

QED. And don't you wish you knew about the Euler identity in high school?